S-wave π-Nucleus Repulsion and Dirac Phenomenology

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Abstract

A relativistic π-nucleon potential is extended to $m^* \neq m$ to investigate the possibility of generating s-wave π-nucleus repulsion. We find that relativity does indeed generate significant repulsion, the exact amount depending on the details of the calculation. In contradistinction the $t\rho$ approximation gives very little repulsion.

The experimental data on low-energy pion-nuclear interactions [1] from a variety of sources, e.g., pionic atoms at zero pion kinetic energy E_{kin}, elastic, inelastic and charge exchange reactions at $E_{\text{kin}} \leq 50$ MeV, etc., are generally well described by theoretically motivated phenomenological optical potentials [2]. These potentials are generated by using the nonrelativistic impulse approximation plus density dependent corrections. Unfortunately they fail to produce the central pion-nucleus repulsion of about 30 MeV required by fits of the experimental data [3].

The underlying isoscalar πN s-wave interaction is exceptionally small at threshold (with a scattering length $b_0 = -0.010 \ m_\pi^{-1}$, m_π being the pion mass), its double-scattering modification [4] is relatively ineffective, and the dispersive contribution accompanying the imaginary πNN absorption term is believed to be attractive [5]; if this latter contribution were to blame, it would have to be *repulsive*, with a magnitude of up to five times its imaginary counterpart.

Recently there has been considerable controversy over the role of relativity in addressing this problem. Birbrair and Gridnev [6, 7] have made the observation that, to the extent that Nuclear Dirac Phenomenology provides a valid and meaningful description of the gross properties of nuclei at low energies, it naturally offers additional s-wave repulsion.

This approach has been attacked from two fronts. First, it has been shown that for pion scattering there are large off-shell ambiguities [8]. Second, it has been argued that for pions the impulse approximation is not valid, since using m^* internally in diagrams cancels most of the m^* effects coming from the impulse approximation [9, 10]. These are both valid concerns.

One way to address these concerns is to take a relativistic model that describes the free π-nucleon scattering and examine its prediction when the nucleon effective mass m^* is varied. The off-shell behavior is then fixed by the model and is no longer ambiguous. In addition m^* is included in all possible places and one goes beyond the impulse approximation. In this letter we use the relativistic π-nucleon potential developed by Pearce and

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Jennings [11] to carry out this program. We find that there is a repulsion on the order of a few tens of MeV coming from the relativistic effects. However, we also find that both objections to the work of Birbrair and Gridnev are correct. The $t\rho$ approximation gives very little dependence on m^* while a significant effect arises from putting m^* in intermediate propagators.

The present work is a first step towards developing a full relativistic optical potential. That work would require the consideration of other partial waves and the treatment of Pauli blocking. A relativistic potential is interesting in its own right and, as pointed out in ref. [12], is crucial in reactions like $(p,n\pi^+)$ where the nucleons are treated most easily in a Dirac approach.

An early attempt to develop a relativistic model was made by Miller and Noble [13]. They used the σ-ω model that did not attempt to simultaneously describe the free nucleon-nucleon scattering. Thus it is quite different from our approach.

An alternate approach to the relativistic pion-nuclear problem has been developed by Leisi and co-workers [14]. This approach introduces an explicit scalar field (σ) for the pion to couple to. In our work we are only concerned with the effects of the pion coupling to an in-medium nucleon and have nothing to say, at the present time, about an explicitly enhanced σ field.

We start with the π-nucleon potential developed by Pearce and Jennings [11]. That potential includes ρ, σ exchange and the nucleon and delta pole and crossed-pole terms. The nucleons are treated as Dirac particles and negative energy propagators are included at each step of the process. In the present work we simply take the Pearce-Jennings model and replace m with m^*. In particular the mass appears in the energy denominators of the nucleon and delta pole and crossed-pole terms and in the free spinors. One has to be careful not to replace m where it has a purely formal role such as in the definition of the pseudo-vector coupling. We have used the smooth propagator [15] and have checked that our results agree with ref. [11] when $m^* = m$.

The calculation of phase shifts uses bare values for the nucleon and delta masses and

couplings of the pole diagrams in the P_{11} and P_{33} channels. These are calculated by the renormalisation procedure described in ref. [16] wherein the P_{11} amplitude is required to have a pole at the physical nucleon mass m, and the residue is required to reproduce the physical πNN coupling constant. The bare delta mass and coupling constant are treated as parameters and adjusted to fit the experimental phase shifts.

The values to use for the in-medium πNN and $\pi N\Delta$ couplings are ambiguous. The obvious thing to do is to keep the bare couplings fixed. The renomalisation will then occur due to the dressing generated by solving the Lippmann-Schwinger equation. This will work well in the p-waves where the renormalisation is generated directly in this manner in free space. In this case the physical couplings will vary with the effective mass m^*.

For the s-waves the physical couplings are put in by hand rather than being generated within the model. It is not clear how they will change in the medium. A possible constraint arises from the Goldberger-Treiman relation. It has been suggested that in-medium we have two Goldberger-Treiman relations [17] depending on which quantum numbers we are considering. For the s-wave the renormalisation is different from that for the p-waves and is quite small [17]. This suggests that one keep the couplings fixed at their physical values. We will give results for both cases.

In our work we chose m^* as the (variable) physical nucleon mass and obtained the corresponding bare nucleon mass by the renormalisation procedure. If we keep the bare coupling constants fixed at the value for $m^* = m$ the renomalised values vary with m^*. The values of the dressed coupling constant and the bare nucleon mass obtained this way are shown in table 1.

To obtain the dressed delta parameters as a function of m^*, we set the dressed delta-nucleon mass difference m^*_Δ-m^* equal to the *physical* mass difference. Calculations were then performed for the P_{33} channel at energies around resonance ($\delta_{P_{33}} = 90^o$). The value of the dressed delta mass was then extracted from the location of the resonance, and that of the dressed coupling constant $f_{\pi N\Delta}$ from the slope of the phase shift curve as a

function of energy. Values of the extracted parameters are shown in table 2.

The results for the scattering lengths in each isospin channel are given in table 3 for three cases: fixed bare couplings, fixed physical couplings and the $t\rho$ approximation. For each case, we also give the isoscalar pseudopotentials [18] obtained from the scattering lengths, using a nuclear matter density of 0.17 fm^{-3}.

It is instructive to examine, for a fixed pion energy, the contributions to the on-shell Born term [11] from the individual diagrams in each isospin channel as a function of m^*. These are shown in figs. 1 and 2 for the case of fixed bare couplings, with $T_\pi = 20$ MeV. We see that the main variation is in the pole and crossed-pole terms. The variation has two main sources. First, the energy denominator varies approximately as $1/m^*$. Second, the couplings vary (when we keep the bare couplings fixed).

Next, we note that in the $t\rho$ approximation there is very little m^* dependence. This is consistent with the off-shell extrapolation used by Gal *et al.* [8] but inconsistent with the extrapolation of Birbrair *et al.* [6, 7]. Thus we conclude that the $t\rho$ approximation cannot account for the repulsion.

We see, however, that including m^* also in the interior of diagrams does give repulsion. This comes about from the energy denominators of the z-graphs. The most repulsion arises when we keep the bare coupling constants fixed and gives about 20 MeV repulsion at nuclear matter density. When the dressed couplings are kept fixed at their physical values (which we believe is more realistic) the repulsion is cut almost in half. Thus we do not have enough repulsion to cure the entire problem but the effect is in the correct direction and within a factor of 2 of the correct size.

In summary, we have examined the predictions of a relativistic π-nucleon potential when $m^* \neq m$ for two different choices of the in-medium coupling constants, and compared them with that of the $t\rho$ approximation. We have found that relativity does indeed generate significant repulsion, the exact amount depending on the choice of the in-medium couplings. In contradistinction the $t\rho$ approximation is found to give very little repulsion.

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Figure Captions

Fig. 1. The on-shell Born potential in the S11 channel (solid line) as a function of m^*/m, for $T_\pi = 20$ MeV and fixed bare couplings. The contributions from the individual diagrams are labelled as follows: (a): nucleon pole; (b): crossed nucleon pole; (c): ρ exchange; (d): Δ pole; (e): crossed Δ pole; (f): σ exchange.

Fig. 2. Same as in fig. 1 but for the S31 channel.

Table 1. The values of the dressed πNN coupling constant and the bare nucleon mass for a range of m^*.

m^*/m	$g^2_{\pi NN}/(4\pi)$	m_{oN} (MeV)
0.5	19.85	652.4
0.6	19.07	739.9
0.7	18.01	827.7
0.8	16.79	915.8
0.9	15.53	1004.3
1.0	14.30	1093.2
1.1	13.16	1182.7
1.2	12.14	1272.7

Table 2. Values of the dressed $\pi N\Delta$ coupling constant and Δ mass for a range of m^*.

m^*/m	$f^2/(4\pi)$	m^*_Δ (MeV)	Γ_Δ (MeV)
0.5	0.496	826.2	169
0.6	0.538	893.1	173
0.7	0.522	968.8	161
0.8	0.475	1052.3	144
0.9	0.415	1140.7	128
1.0	0.365	1231.8	113
1.1	0.308	1324.4	102
1.2	0.270	1417.6	92

Table 3. Scattering lengths and pseudopotential strengths for a range of m^*.

m^*/m	In Medium Couplings			Free Couplings			$t\rho$ Approximation		
	a_{S11} (fm)	a_{S31} (fm)	V (MeV)	a_{S11} (fm)	a_{S31} (fm)	V (MeV)	a_{S11} (fm)	a_{S31} (fm)	V (MeV)
0.5	0.108	-0.184	26.2	0.134	-0.146	16.0	0.226	-0.123	2.1
0.6	0.118	-0.183	25.0	0.150	-0.142	13.4	0.222	-0.121	2.0
0.7	0.140	-0.169	20.1	0.167	-0.136	10.6	0.218	-0.119	2.0
0.8	0.166	-0.151	13.8	0.183	-0.129	7.6	0.216	-0.118	2.0
0.9	0.191	-0.132	7.3	0.199	-0.122	4.7	0.214	-0.117	2.0
1.0	0.213	-0.116	2.0	0.213	-0.116	2.0	0.213	-0.116	2.0
1.1	0.230	-0.103	-2.4	0.225	-0.110	-0.5	0.211	-0.115	1.9
1.2	0.243	-0.093	-5.7	0.235	-0.105	-2.6	0.210	-0.115	1.9